

Topics in High-Dimensional Probability and Statistics*

Homework 1

Exercise 1

Let $X \in SG(\sigma^2)$ (X is sub-gaussian random variable with sub-gaussian parameter σ^2). Show that $\text{Var} X \leq \sigma^2$. Therefore σ^2 might be called "variance proxy".

Exercise 2

Let $X \in SG(\sigma_1^2)$, $Y \in SG(\sigma_2^2)$. X and Y are not necessary independent. Show that $X + Y \in SG((\sigma_1 + \sigma_2)^2)$.

Exercise 3

Prove that X is sub-gaussian random variable if and only if $\exists c > 0$ such that $\forall t > 0$:

$$\mathbb{P}\{|X - \mathbb{E}X| > t\} \leq 2e^{-ct^2}$$

Try to find the relation between c and sub-gaussian parameter σ^2 .

Exercise 4

Prove that X is sub-exponential random variable if and only if $\exists c_1, c_2 > 0$ such that $\forall t > 0$:

$$\mathbb{P}\{|X - \mathbb{E}X| > t\} \leq c_1 e^{-c_2 t}$$

Try to find the relation between c_1 , c_2 and sub-exponential parameters σ , b .

Exercise 5

Let ξ be random variable with density $p_\xi(x) = x^{-2}$ for $x \geq 1$ and 0 otherwise. Is ξ sub-gaussian? Sub-exponential?

Exercise 6

Let X be centered sub-gaussian random variable, $\lambda \in \mathbb{R}$. Show that $\|\lambda X\|_{\Psi_2} = |\lambda| \|X\|_{\Psi_2}$.

Exercise 7

Let X and Y be centered sub-gaussian variables. Show that $\|X + Y\|_{\Psi_2} \leq \|X\|_{\Psi_2} + \|Y\|_{\Psi_2}$.

*Teaching material can be found at <https://www.qparis-math.com/teaching>.