

MMDA SEMINAR 2

1. RECAP

Make sure you can shortly answer the following questions.

1. What is a measurable function ?
2. Give basic examples.
3. What do we call a simple function ?
4. What is the basic approximation property of positive simple functions with respect to positive measurable functions ?

2. IMAGE MEASURE

Let (S, \mathcal{S}, μ) be a measure space. Let (T, \mathcal{T}) be a measurable space and $f : S \rightarrow T$ be a measurable function. Define $\mu_f : \mathcal{T} \rightarrow [0, +\infty]$, for all $B \in \mathcal{T}$, by

$$\mu_f(B) := \mu(f^{-1}(B)).$$

1. Show that μ_f is a positive measure.
2. Show that μ_f and μ have same total mass.
3. Suppose that μ is a discrete measure. What can you say about μ_f ?
3. Suppose that $(T, \mathcal{T}) = (\mathbb{R}, \mathcal{B}(\mathbb{R}))$ and that $f : S \rightarrow \mathbb{R}$ is a simple function. What can you say about μ_f ?
4. Suppose that μ_f is a σ -finite measure. What can you say about μ ?